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FIXED POINT THEOREMS ON CONTROLLED METRIC SPACES

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ABSTRACT

In this paper,obtainedunique fixed point theorems ona controlledmetric spaces.Which is generalize the results of Kiran et al.[24]and many others results.

KEYWORDS: *Fixed Point, Controlled Metric Space, b-Metric Space, Extended b-Metric Space*

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INTRODUCTION

The well- known Banach contraction theorem [1] has been generalized and extended by many authors (see [2]- [8]). Bakhtin [9],Bourbaki [10] and Czerwik [11, 12] introduced the concept of b- metric space. After that, a number of research papers have been established that generalized that Banach fixed point result in the framework of b- metric space (see [13]- [18]). Kamran et al.[23] generalized the structure of a b- metric space and called it an extended b - metric space. Thereafter, many research article have appeared, which generalize the contraction principle of Banach in extended b- metric space (see[19], [20], [21], [22],[24]). Mlaiki et al. [25] generalized the structure ofextended b - metric space and called a controlled metric. In this structure many authors obtained fixed point theorem, which is generalize the contraction principle of a Banach in controlled metric space (see [26]).In this paper, obtain a unique fixed point theorem and example a controlled metric space, which is generalize a number of fixed point results of Kiran et al. [24] and others.

PRELIMINARIES

Definition 2.1 [11] LetX be a set and s ≥1 a real number. A function $d_\theta : X \times X \to [0, \infty)$ is called a b - metric space, if it satisfies the following axioms for all x, y, z ε X.

- $d_\theta(x, y) = 0$ if and only if$x = y$,
- $d_\theta(x, y) = d_\theta(y, x)$,
- $d_\theta(x, y) \le s[d_\theta(x, z) + d_\theta(z, y)]$. The pair (X, d_θ) is called a b - metric space.Clearly, every metric space is a b - metric space with s = 1, but its converse is not true in general.

Definition 2.2 [23]Let X be a non- empty set and$\alpha : X \times X \to [1, \infty)$.A function $d_\Phi : X \times X \to [0, \infty)$ is called an extendedb - metric space, if it satisfies the following axioms for all x, y, z ε X.

- $d_\Phi(x, y) = 0$ if and only if$x = y$,
- $d_\Phi(x, y) = d_\Phi(y, x)$,

- $d_\Phi(x, y) \leq \alpha (x, y) [d_\Phi (x, z) +d_\Phi(z, y)]$.The pair (X,d_Φ) is called an extendedb - metric space.

Example 2.1 [23]Let $X = [0, \infty)$. Define $d_\Phi : X \times X \to [0, \infty)$

$$d_\Phi(x, y)= \begin{cases} 0, & \text{if } x=y \\ 3, & \text{if } x \text{ or } y \, \varepsilon \{1,2\} \\ 5, & \text{if } x \neq y \, \varepsilon \{1,2\} \\ 1, & \text{otherwise.} \end{cases}$$

Then(X, d_Φ) isextended b - metric space, where$\alpha : X \times X \to [1, \infty)$ is defined by $\alpha(x, y) =x + y + 1$, for all x, y ε X.Every b - metric space is an extended b – metricspace with constant function $\alpha(x, y) = s$ for $s\geq 1$, but its converse is not in general.

Definition 2.3 [25] Let X be a non- empty set and$\alpha : X \times X \to [1, \infty)$.A function $d : X \times X \to [0, \infty)$ is calledcontrolledmetric space, if it satisfies the following axioms for all x, y, z ε X.

- $d (x, y) =0$ if and only if$x = y$,
- $d (x, y) =d (y, x)$,
- $d (x, y) \leq \alpha (x, z)d (x, z) + \alpha (z, y) d (z, y)]$.The pair (X,d) is called an controlledmetric space.

Example 2.2 [25] Let X = { 0, 1, 2 }. Consider the function $d : X \times X \to [0, \infty)$ defined by

$d (0, 0) = d (1, 1) = d (2, 2) = 0$, $d (0, 1) = d (1, 0) = 1$,$d (0, 2) = d (2, 0) = 1/2$,$d (1, 2) = d (2, 1) = 2/5$.Take $\alpha : X \times X \to [1, \infty)$ to be symmetry and be defined by

$\alpha (0, 0)=\alpha (1, 1)=\alpha (2, 2)= \alpha (0, 2) = \alpha (2, 0) = 1$, $\alpha (0, 1)= \alpha (1, 0)=11/10$,$\alpha (1, 2) = \alpha (2, 1)= 5/4$.

It is easy to show that d is a controlled metricspace.

Note that

$d (0, 1) = 1 > 99/100 = \alpha (0, 1) [d (0, 2) + d (2, 1)]$.

Thusdis not an extended b - metric for the same function α.

Theorem 2.1 [24] Let (X, d_Φ) be a complete extended b - metricspace with $\alpha : X \times X \to [1, \infty)$. IfT: $X \to X$ satisfies the inequality,

$d_\Phi (Tx, Ty)\leq a d_\Phi (x, y) + b\, d_\Phi (x,, Tx) +c\, d_\Phi (y, Ty) + e [d_\Phi (x, Ty) +d_\Phi (y, Tx)]$.

Where a, b, c, e $\geq$ 0 and for eachx_0 ε X,

$a + b + c +2e \lim_{n, m \to \infty} \alpha (x_n, x_m)< 1$. Then T has a fixed point.

Theorem 2.2 [24]Let (X, d_Φ) be a complete extended b - metricspace with $\alpha : X \times X \to [1, \infty)$IfT : $X \to X$ satisfies the inequality,

$d_\Phi (Tx, Ty)\leq a\, d_\Phi (x, y) + b [d_\Phi (x,, Tx) +d_\Phi (y, Ty)$for each x, y ε X, where a, b ε [0, 1/3). Moreover for eachx_0 ε X,$\lim_{n, m \to \infty} \alpha (x_n, x_m)$ b< 1.Then Thas aunique fixed point.

Lemma 2.1 For every sequence $\{x_n\}_{n \in N}$ of elements from a controlled metricspace (X, d)the inequality

$$d(x_n,x_m) \le \alpha(x_n,x_{n+1}) d(x_n,x_{n+1}) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} \beta(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_i,x_{i+1}) + \prod_{k=n+1}^{m-1} \beta(x_k,x_m) d(x_{m-1},x_m)$$

Proof -$d(x_n, x_m) \le \alpha(x_n, x_{n+1}) d(x_n, x_{n+1}) + \alpha(x_{n+1}, x_m) d(x_{n+1}, x_m)$

$\le \alpha(x_n, x_{n+1}) d(x_n, x_{n+1}) + \alpha(x_{n+1},x_m) \alpha(x_{n+1}, x_{n+2}) d(x_{n+1}, x_{n+2}) + \alpha(x_{n+1}, x_m) \alpha(x_{n+2}, x_m) d(x_{n+2}, x_m)$

$\le \alpha(x_n, x_{n+1}) d(x_n, x_{n+1}) + \alpha(x_{n+1}, x_m) \alpha(x_{n+1}, x_{n+2}) d(x_{n+1}, x_{n+2}) + \alpha(x_{n+1}, x_m) \alpha(x_{n+2}, x_m) \alpha(x_{n+2}, x_{n+3}) d(x_{n+2}, x_{n+3})$

$+\alpha(x_{n+1}, x_m) \alpha(x_{n+2}, x_m) \alpha(x_{n+3}, x_m) d(x_{n+3}, x_m)$

$\le \dots$

$\le \alpha(x_n, x_{n+1}) d(x_n, x_{n+1}) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} \alpha(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_i,x_{i+1}) + \prod_{k=n+1}^{m-1} \alpha(x_k,x_m) d(x_{m-1},x_m)$.

Hence

$$d(x_n,x_m) \le \alpha(x_n,x_{n+1}) d(x_n,x_{n+1}) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} \alpha(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_i,x_{i+1}) + \prod_{k=n+1}^{m-1} \alpha(x_k,x_m) d(x_{m-1},x_m). \quad (1)$$

Lemma 2.2 Every sequence $\{x_n\}_{n \in N}$ of elements from a controlled metric space (X, d),having the property that there existskϵ[0, 1) such that

$$d(x_{n+1}, x_n) \le k d(x_n, x_{n-1}), \quad (2)$$

for every n ε N is Cauchy sequence.

Proof – First, by successively applying. (2), we get

$$d(x_{n+1}, x_n) \le k^n d(x_1, x_0) \quad (3)$$

for every kεN.

Then by lemma (2.1), for all n, m ε N, we have

$d(x_n, x_m) \le \alpha(x_n, x_{n+1}) d(x_n, x_{n+1}) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i(x_i,x_{i+1})d(x_i,x_{i+1}) + \prod_{k=n+1}^{m-1} \alpha(x_k,x_m) d(x_{m-1},x_m),} \alpha(x_j,x_m)\right) \alpha d(x_n, x_m)$

$\le \alpha(x_n, x_{n+1}) d(x_n, x_{n+1}) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} \alpha(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_i,x_{i+1}) +$

$\prod_{k=n+1}^{m-1} \alpha(x_k,x_m) d(x_{m-1},x_m), \alpha(x_{m-1}, x_m)$

$d(x_n, x_m) \le \alpha(x_n, x_{n+1}) k^n d(x_0, x_1) + \sum_{i=n+1}^{m-2} \left(\prod_{j=n+1}^{i} \alpha(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_0,x_1) k^i +$

$\prod_{k=n+1}^{m-1} \alpha(x_k,x_m) d(x_0,x_1), \alpha(x_{m-1}, x_m) k^{m-1}$

$d(x_n, x_m) \le \alpha(x_n, x_{n+1}) k^n d(x_0, x_1) + \sum_{i=n+1}^{m-1} \left(\prod_{j=n+1}^{i} \alpha(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_0,x_1) k^i \quad (4)$

$$\text{Let} S_1 = \sum_{i=0}^{i} \left(\prod_{j=0}^{i} \alpha(x_j,x_m)\right) \alpha(x_i,x_{i+1}) d(x_0,x_1) k^i.$$

From 2.4, we get

$$d(x_n, x_m) \le \alpha(x_n, x_{n+1}) [\, k^n d(x_0, x_1) + S_{m-1} - S_n \,]. \quad (5)$$

As above, using$\alpha(x, k) \ge 1$, and ratio test,

$\lim_{n \to \infty} S_n$ exists. Thus $\{S_n\}$ is Cauchy. Finally,letting n, m $\to \infty$in 5, we conclude that

$\lim_{n,m\to\infty} d(x_n, x_m)=0$. Thus $\{x_n\}_{n \varepsilon N}$ is a Cauchy sequence.

MAIN RESULTS

Theorem 3.1 Let (X, d) be a complete controlled metricspaces with $\alpha: X \times X \to [1, \infty)$. If $T: X \to X$ satisfies the inequality

$$d(Tx, Ty) \le a d(x, y) + b d(x, Tx) + c d(y, Ty) + e[d(x, Ty) + d(y, Tx)] \quad (6)$$

Where a, b, c, e ≥ 0 and for each $x_0 \varepsilon X$,

$a + b + c + 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) < 1$. Then T has a fixed point.

Proof–Let us choose an arbitrary $x_0 \varepsilon X$ and define the iterative sequence $\{x_n\}_{n \varepsilon N}$ by

$x_n = Tx_{n-1} = T^{n-1}x_0$ for all $n \ge 1$.

If $x_n = x_{n-1}$, then x_n is a fixed point of T and the proof holds. So, we suppose

$x_n \neq x_{n-1}$, for all $n \ge 1$. Then from equation 6, we have

$d(x_{n+1}, x_n) = d(Tx_n, Tx_{n-1})$

$\le a d(x_n, x_{n-1}) + b d(x_n, Tx_n) + c d(x_{n-1}, Tx_{n-1}) + e[d(x_n, Tx_{n-1}) + d(x_{n-1}, Tx_n)]$

$\le a d(x_n, x_{n-1}) + b d(x_n, Tx_n) + c d(x_{n-1}, Tx_{n-1}) + e[d(x_n, x_n) + d(x_{n-1}, Tx_n)]$

$\le a d(x_n, x_{n-1}) + b d(x_n, Tx_n) + c d(x_{n-1}, Tx_{n-1}) + e d(x_{n-1}, Tx_n)$

$\le a d(x_n, x_{n-1}) + b d(x_n, Tx_n) + c d(x_{n-1}, Tx_{n-1}) + e[\alpha(x_{n-1}, x_n) d(x_{n-1}, x_n) + \alpha(x_n, Tx_n) d(x_n, Tx_n)]$

$\le [a + c + e\alpha(x_{n-1}, x_n)] d(x_{n-1}, x_n) + [b + e\alpha(x_n, Tx_n)] d(x_n, Tx_n)$

$= [a + c + e\alpha(x_{n-1}, x_n)] d(x_{n-1}, x_n) + [b + e\alpha(x_n, x_{n+1})] d(x_n, x_{n+1}) \quad (7)$

Similarly,

$$d(x_n, x_{n+1}) \le [a + b + e\alpha(x_{n-1}, x_n)] d(x_{n-1}, x_n) + [c + e\alpha(x_n, x_{n+1})] d(x_n, x_{n+1}) \quad (8)$$

Adding(7)and (8), we get

$2d(x_n, x_{n+1}) \le [2a + b + c + 2e\alpha(x_{n-1}, x_n)] d(x_{n-1}, x_n) + [b + c + 2e\alpha(x_n, x_{n+1})] d(x_n, x_{n+1})$

$[2 - b - c - 2e\alpha(x_n, x_{n+1})] d(x_n, x_{n+1}) \le [2a + b + c + 2e\alpha(x_{n-1}, x_n)] d(x_{n-1}, x_n)$

$d(x_n, x_{n+1}) \le [2a + b + c + 2e\alpha(x_{n-1}, x_n)] d(x_{n-1}, x_n) / [2 - b - c - 2e\alpha(x_n, x_{n+1})] d(x_n, x_{n+1}) \le \mu d(x_{n-1}, x_n)$ where, $\mu = [2a + b + c + 2e\alpha(x_{n-1}, x_n)] / [2 - b - c - 2e\alpha(x_n, x_{n+1})]$

Since, $a + b + c + 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) < 1$,

$2a + 2b + 2c + 4e \lim_{n,m\to\infty} \alpha(x_n, x_m) < 2$,

$2a + 2b + 2c + 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) + 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) < 2$,

$2a + b + c + 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) < 2 - b - c - 2e \lim_{n,m\to\infty} \alpha(x_n, x_m)$,

$2a + b + c + 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) / 2 - b - c - 2e \lim_{n,m\to\infty} \alpha(x_n, x_m) < 1$.

Implies, $\mu < 1$.

Hence from lemma 2.2, $\{ x_n \}_{n \varepsilon N}$is a Cauchy sequence. As X is complete, therefore there existsx ε X

such that $\lim_{n\to\infty} x_n = x$. Next, we will show that x is a fixed point of T. From the triangle inequality and equation (6), we have

$d(x, Tx) \leq \alpha(x, x_{n+1}) d(x, x_{n+1}) + \alpha(x_{n+1}, Tx)\, d(x_{n+1}, Tx)$

$\leq \alpha(x, x_{n+1}) d(x, x_{n+1}) + \alpha(x_{n+1}, Tx)\, [a\, d(x_n, x) + b\, d(x_n, Tx_n) + c\, d(x, Tx) + e\, [d(x_n, Tx) + d(x, Tx_n)]$

$\leq \alpha(x, x_{n+1}) d(x_n, x_{n+1}) + a\, \alpha(x_{n+1}, Tx) d(x_n, x) + b\alpha(x_{n+1}, Tx) d(x_n, x_{n+1}) + c\alpha(x_{n+1}, Tx)\, d(x, Tx) + e\, \alpha(x_{n+1}, Tx)\, d(x, x_{n+1}) + e\alpha(x_{n+1}, Tx)[\alpha(x_n, x)\, d(x_n, x) + \alpha(x, Tx)\, d(x, Tx)]$

$= [\alpha(x, x_{n+1}) + b\, \alpha(x_{n+1}, Tx) + e\alpha(x_{n+1}, Tx)]\, d(x, x_{n+1}) + [a\alpha(x_{n+1}, Tx) + e\, \alpha(x_{n+1}, Tx)\alpha(x_n, x)]\, d(x_n, x) + [c\alpha(x_{n+1}, Tx) + e\alpha(x_{n+1}, Tx)\alpha(x_n, x)] d(x, Tx)[1 - c\, \alpha(x_{n+1}, Tx) - e\alpha(x_{n+1}, Tx)\alpha(x, Tx)] d(x, Tx)$

$\leq [\alpha(x_{n+1}, Tx) + b\alpha(x_{n+1}, Tx) + e\alpha(x_{n+1}, Tx)] d(x, x_{n+1}) + [a\alpha(x_{n+1}, Tx) + e\alpha(x_{n+1}, Tx)\alpha(x_n, x)] d(x, Tx) \to 0$ as $n \to \infty$.

$[1 - c\, \alpha(x_{n+1}, Tx) - e\alpha(x_{n+1}, Tx)\alpha(x, Tx)]\, d(x, Tx) \leq 0.$ (10)

Similarly, $[1 - b\, \alpha(Tx, x_{n+1}) - e\, \alpha(Tx, x_{n+1})\alpha(Tx, x)]\, d(x, Tx) \leq 0$ (11)

Adding (10)and (11), we have

$[2 - b\, \alpha(Tx, x_{n+1}) - c\, \alpha(x_{n+1}, Tx) - 2\, e\, \alpha(x_{n+1}, Tx)\alpha(x, Tx)]\, d(x, Tx) \leq 0.$

Since, $[2 - b\, \alpha(Tx, x_{n+1}) - c\, \alpha(x_{n+1}, Tx) - 2\, e\, \alpha(x_{n+1}, Tx)\alpha(x, Tx)] > 0$,

We get $d(x, Tx) = 0$ ie $Tx = x$.

Now, we show that x is the unique fixed point of T. Assume y is another fixed point of T, then we have $Ty = y$. Also,

$d(x, y) = d(Tx, Ty) \leq a\, d(x, y) + b\, d(x, Tx) + c\, d(y, Ty) + e\, [d(x, Ty) + d(y, Tx)]$

$\leq a\, d(x, y) + b\, d(x, x) + c\, d(y, Ty) + e\, [d(x, y) + d(y, x)]$

$\leq a\, d(x, y) + 2e\, d(x, y)$

$[1 - a - 2e]\, d(x, y) \leq 0.$

As, $a + b + c + 2e \leq a + b + c + 2e \lim_{n, m\to\infty} \alpha(x_n, x_m) < 1$.

Therefore, $[1 - a - 2e] > 0$, and $d(x, y) = 0$ ie $x = y$.

Hence T has a unique fixed point in X.

Remark 3.1 From the symmetry of the distance function d, it is easy to prove similar to that in [4, 14]that b = c. Thus the inequality (6) is equivalent to the following inequality

$d(Tx, Ty) \leq a\, d(x, y) + b\, [d(x, Tx) + d(y, Ty)] + e\, [d(x, Ty) + d(y, Tx)]$ (12)

where $a, b, e \geq 0$ such that $a + 2b + 2e \lim_{n, m\to\infty} \alpha(x_n, x_m) < 1$.

If a = b = 0 and e ε [0, ½) in equality (12), we obtain generalization of Chatterjee's maps [8] in controlled metric space.

Remark 3.2 Theorem 3.1 generalizes and improves Theorem 9 of [16] and therefore Theorem 2.1 of [3]. Moreover,Theorem 3.1 generalizes and improves Theorem12 of [21],Theorem 2.19 from [28] and Theorem 9 from [24].

Theorem 3.2 Let (X, d) be a complete controlled metric space with $\alpha: X \times X \to [1, \infty)$. If $T: X \to X$ satisfies the inequality

$$d(Tx, Ty) \leq a d(x, y) + b[d(x,Tx) + d(y, Ty)] \quad (13)$$

for each x, y ε X, where a, b ε [0, 1/3). Moreover for each x_0 ε X.

$\lim_{n, m\to\infty} \alpha(x_n, x_m) b < 1$. Then T has a fixed point.

Proof–Let us choose an arbitrary x_0 ε X and define the iterative sequence $\{x_n\}_{n \varepsilon N}$ by

$x_n = Tx_{n-1} = T^{n-1}x_0$ for all $n \geq 1$.

If $x_n = x_{n-1}$, then x_n is a fixed point of T and the proof holds. So, we suppose

$x_n \neq x_{n-1}$, for all $n \geq 1$. Then from equation 3.8, we have

$d(Tx_n, Tx_{n-1}) \leq a d(x_n, x_{n-1}) + b[d(x_n,Tx_n) + d(x_{n-1}, Tx_{n-1})] d(x_{n+1}, x_n)$

$\leq a d(x_n, x_{n-1}) + b[d(x_n, x_{n+1}) + d(x_{n-1}, x_n)]$

$[1 - b] d(x_{n+1},x_n) \leq [a+ b] d(x_n, x_{n-1})$

$d(x_{n+1},x_n) \leq [a+ b] d(x_n, x_{n-1}) / [1 - b]$

$$\leq \mu d(x_n, x_{n-1}) \quad (14)$$

Where $\mu = \{a + b\} / \{1 - b\}$.

Since a, b ε [0, 1/3), so $\mu < 1$.Hence from lemma (6), $\{x_n\}_{n \varepsilon N}$ is a Cauchy sequence. As X is complete, therefore there exists x ε X such that $\lim_{n\to\infty} x_n = x$. Next, we will show that x is a fixed point of T. From the triangle inequality and equation 3.8, we have

$d(x, Tx) \leq \alpha(x, x_{n+1}) d(x, x_{n+1}) + \alpha(x_{n+1}, Tx) d(x_{n+1}, Tx)$

$\leq \alpha(x, x_{n+1}) d(x, x_{n+1}) + \alpha(x_{n+1},Tx) \{a d(x,x_n) + b[d(x,Tx) + d(x_n, Tx_n)]\} [1- b\alpha(x_{n+1},Tx)] d(x, Tx)$

$\leq \alpha(x_{n+1},x) d(x, x_{n+1}) + a \alpha(x_{n+1},Tx) d(x, x_n) + b\alpha(x_{n+1},Tx)] d(x_n, Tx_n) \to 0$ as $n \to \infty$.

$$[1- b\alpha(x_{n+1},Tx)] d(x, Tx) \leq 0. \quad (15)$$

Since, $\lim_{n, m\to\infty} \alpha(x_n, x_m) b < 1$.

We get

$[1 - b\alpha(x_{n+1},Tx)] > 0$ and so

$d(x, Tx) = 0$ ie $Tx = x$.

Now, we show that x is the unique fixed point of T. Assume y is another fixed point of T, then we have $Ty = y$. Also,

$d(x, y) = d(Tx, Ty) \leq a d(x, y) + b[d(x,Tx) + d(y, Ty)]$

$\leq a d(x, y) < d(x, y)$.

Which is a contradiction Henced (x, y)=0iex=y. Hence T has a unique fixed point in X.

Remark 3.3 Theorem 3.2 generalizes Theorem2 of [13] and Theorem 10 of [24].

Example 3.1 Let X = {0, 1, 2}. Consider the function d: X × X → [0, ∞) defined by

d (0, 0) = d (1, 1) = d (2, 2) = 0, d (0, 1) = d (1, 0) = 10,d (0, 2) = d (2, 0) = 5,d (1, 2) = d (2, 1) = 30.

Take α : X × X → [1, ∞) to be symmetry and be defined by

α (0, 0)=α (1, 1)=α (2, 2)= α (0,1) = α (1, 0) = 1, α (0, 2)= α (2, 0)=4,α (1, 2) = α (2, 1)= 1.

It is easy to show that d is a controlled metric space.

Note that,

d(1, 2) = 30 >15 = α (1, 2) [d (1, 0) + d (0, 2)].

Thusdis not an extended b - metric space.SupposefunctionT: X → Xsuch that

T0 =0, T2 =0andT1=2.If a = 3/15, b = 1/15, c= 2/15 and e=1/15.

Hence all the condition of Theorem3.1 are satisfies and so T has a unique fixed point isx = 0.

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